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RISK-BASED GROSS PREMIUM ANALYSIS FOR PENSION LIFE INSURANCE

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ABSTRACT

Pension life insurance provides financial protection to the insured when they reach pension age or to the beneficiaries if the insured dies before reaching pension age. In order to give this protection, insurance companies must determine premium carefully, taking into account the level of risk and uncertainty of potential claims. This study aims to develop a risk-based gross premium calculation model for pension life insurance using variance and portfolio approaches, while evaluating the effect of portfolio size on the relative risk load. The method used is actuarial analysis using the Indonesian Mortality Table (TMI) 2019. The model is designed considering two scenarios: the insured survives until retirement age and the insured dies before reaching retirement age. The results of the study show that the gross premium model can be explicitly stated by considering the expected value of benefits and the standard deviation of claims. The gross premium is obtained by adding the relative risk load to the net premium. Additionally, the larger the size of the insured group in the portfolio, the lower the relative risk load. The gross premium value also approaches the net premium value as the number of insured individuals increases.

Keywords: *Pension life insurance, risk-based gross premium, variance, portfolio, relative risk load*

1. INTRODUCTION

Every individual faces various risks in their life. As a worker, the risks encountered can vary widely, such as work accidents, disability, illness, and death (Nasution, 2020), as well as income uncertainty when approaching pension age. One way to anticipate these risks is by participating in an insurance program, which is a financial instrument that can provide protection from various unexpected risks (Fauzi, 2019). The agreement spe cifies the premium amount, which is determined through actuarial calculations to ensure fairness and affordability for the insured (Zoelanda et al., 2017). Therefore, careful analysis is needed when setting premiums.

The risk premium constitutes the fundamental component of premium determination before the inclusion of additional loadings and expenses. These premiums cover only the expected claims from the risk without any extra profit (Dickson, 2005). To deal with this uncertainty, insurance companies need to add a risk load based on variance and standard deviation because both provide a clear and measurable way to see the spread of claims, so the set premium can reflect the risk faced and handle claim fluctuations.

One strategy for reducing claim variability is portfolio pooling, which means gathering various insurance policies with similar characteristics and risks (Dona, 2024). With portfolio pooling, claim fluctuations can be diversified across a larger group of policyholders, so the overall variance and standard deviation become lower. This, in turn, will reduce the relative risk load and allow insurance companies to set premiums that are affordable for policyholders while still profitable for the company. This portfolio pooling strategy can be applied to different types of insurance, including life insurance, by adjusting the characteristics of the products being managed.

Mualem and Zaks (2019) constructed a mathematical model for calculating insurance premiums by taking into account risk and variability in whole life insurance, endowment life insurance, and pension insurance. For pension insurance, they built a premium calculation model where pension benefits are paid only if the insured is still alive at pension age. As a result, it was found that combining portfolios with similar types of risk under one management (whether through acquisition, administrative merger, or other forms of collaboration) can significantly reduce the risk load and improve the financial stability of the company.

This study aims: (1) to develop a risk-based gross premium model for pension life insurance using variance and portfolio approaches; and (2) to evaluate the effect of portfolio size on the relative risk load. Pension life insurance is an insurance product that combines two components: life insurance and a pension fund. Under this arrangement, the policyholder receives retirement benefits accumulated during the working period. Life protection is provided to the beneficiaries if the insured dies before reaching retirement age, like death before pension age (Taim, 2023). In return for this protection, the policyholder is required to pay a certain premium to the insurance company:

According to Dickson et al. (2009), a premium is the amount of money that the insured must pay to the insurance company in exchange for the benefits promised by the insurance company if an event specified in the insurance policy occurs. Premiums are commonly classified into two categories: net premiums and gross premiums. A net premium excludes insurer expenses, profit margins, and other loadings. The net premium can be calculated by applying the equivalence principle, which is the principle stating that the expected present value of the premiums paid must be equal to the expected present value of the benefits to be received. The equivalence principle for the net premium implies that:

$$EPV\ of\ benefit\ outgo = EPV\ of\ net\ premium\ income \quad (1)$$

In this study, the gross premium is defined as the net premium plus a risk loading that reflects claim uncertainty. According to Futami (1994), the gross premium can be calculated using:

$$P^* = P(1+z) \quad (2)$$

Equation (2) can be modified to obtain a gross premium model that explicitly incorporates claim uncertainty as follows:

$$P^* = P(1+\delta^*) \quad (3)$$

To derive the relative risk load, the risk load is first defined. Following Mualem and Zaks (2019), the risk load is assumed to be three times the standard deviation (3σ), providing a safety margin against claim variability:

$$\delta = EPV(GP) - EPV(NP) = 3\sigma \quad (4)$$

The risk load is then added to the net premium to obtain the risk-based gross premium. Next, the relative risk load is defined as the ratio between the risk load and the net premium, which represents the proportion of the net premium required as compensation for claim uncertainty. Mathematically, it's written as (Mualem and Zaks, 2019):

$$\delta^* = \frac{E[\text{PV premi bruto}] - E[\text{PV premi neto}]}{E[\text{PV premi neto}]} = 3\tilde{\sigma} \quad (5)$$

Where $$\tilde{\sigma} = \frac{\sigma_A}{A_x} + \frac{\sigma_a}{a_x}$$

In the context of risk measurement involving the average of several random variables, the standard deviation alone is not enough to show how much uncertainty there is in the average value relative to its expected value, especially when the sample size increases. Statistically, it's known that the variance of a sample mean of size n is $\frac{\sigma^2}{n}$, so the standard deviation becomes:

$$\frac{\sigma}{\sqrt{n}} \quad (6)$$

Equation (6) shows that the deviation of the average value from the expected value decreases as the sample size increases. Consequently, the relative variability of the sample mean decreases as the portfolio size increases.

By understanding probability distributions, insurance companies can assess risks more accurately. This concept plays an important role in the context of pension life insurance, since premium calculations and risk loads heavily depends on the chances of individuals living or dying. The probability that a life aged x dies before age r is denoted by ${}_{r-x}q_x$ and is formulated as:

$$ {}_{r-x}q_x = \frac{l_x - l_r}{l_x} \tag{7} $$

so the probability that a person aged x survives for $r - x$ years is expressed as:

$$ {}_{r-x}p_x = 1 - {}_{r-x}q_x \tag{8} $$

The survival probabilities are subsequently used in calculating the expected present value of various types of annuities, especially life annuities. Based on that, consider an annuity with a total amount of 1 per year, payable m times per year in advance over the lifetime of (x), with each payment being $\frac{1}{m}$. Using an indicator random variable, the expected present value for a life annuity paid m times a year can be expressed as:

$$ \ddot{a}_x^{(m)} = \sum_{t=0}^{\infty} \frac{1}{m} v^{\frac{t}{m}} {}_{\frac{t}{m}}p_x \tag{9} $$

Where

$\ddot{a}_x^{(m)}$ denotes the expected present value (EPV) of a whole life annuity-due payable m times per year for a life aged x.

$v = (1+i)^{-1}$ denotes the discount factor.

Equation (9) can be generalized for a person aged r years into:

$$ \ddot{a}_r^{(m)} = \sum_{t=0}^{\infty} \frac{1}{m} v^{\frac{t}{m}} {}_{\frac{t}{m}}p_r \tag{10} $$

Similarly, for a beneficiary aged $y + r - x$

$$ \ddot{a}_{y+r-x}^{(m)} = \sum_{t=0}^{\infty} \frac{1}{m} v^{\frac{t}{m}} {}_{\frac{t}{m}}p_{y+r-x} \tag{11} $$

Where

$\ddot{a}_r^{(m)}$ = Expected present value of a discrete life annuity-due with payable in advance m times per year for (r).

$\ddot{a}_{y+r-x}^{(m)}$ = Expected present value of a discrete life annuity-due with payable in advance m times per year for $(y + r - x)$.

Consider a temporary life annuity-due with a total amount of 1 per year, paid at the beginning of the period m times a year throughout the life of (x) for a maximum term of $r - x$ years, with each payment being $1/m$.

Using indicator random variables, the expected present value of a life annuity for n years, paid m times a year, can be expressed as:

$$ \ddot{a}_{x:\overline{r-x}|}^{(m)} = \sum_{t=0}^{m(r-x)-1} \frac{1}{m} v^{\frac{t}{m}} {}_{\frac{t}{m}}p_x \tag{12} $$

Where

$\ddot{a}_{x:\overline{r-x}|}^{(m)}$ = Expected present value (EPV) of a temporary life annuity-due payable m times per year for $r - x$ years.

Another important type of life annuity is the deferred life annuity, in which payments commence after a specified deferment period. Consider an annuity with a total amount of 1 per year, paid in advance m times per year, starting at age $x + k$, and payable throughout the remaining lifetime of the insured, with each payment being $\frac{1}{m}$. The present value expectation of a lifetime annuity with payments deferred for k years and paid m times per year is given by:

$$ {}_{k|}\ddot{a}_x^{(m)} = v^k {}_k p_x \ddot{a}_{x+k}^{(m)} \tag{13} $$

Where

${}_{k|}\ddot{a}_x^{(m)}$ = Present value expectation of a discrete whole life annuity with payments made at the beginning m times per year, deferred for k years, for (x) at the start of the contract.

Equation (13) can be extended to formulate a deferred annuity for $r - x$ years for a person aged x into:

$$ {}_{r-x|}\ddot{a}_x^{(m)} = v^{r-x} {}_{r-x}p_x \ddot{a}_r^{(m)} \tag{14} $$

Similarly, for a beneficiary aged y

$$ {}_{r-x|}\ddot{a}_y^{(m)} = v^{r-x} {}_{r-x}p_y \ddot{a}_{y+r-x}^{(m)} \tag{15} $$

Where

${}_{r-x|}\ddot{a}_x^{(m)}$ = Present value expectation of a discrete whole life annuity with payments made at the beginning m times per year, deferred for $r - x$ years, for (x) at the start of the contract.

${}_{r-x|}\ddot{a}_y^{(m)}$ = Present value expectation of a discrete whole life annuity with payments made at the beginning m times per year, deferred for $r - x$ years, for (y) at the start of the contract.

2. METHODOLOGY

This study uses secondary data obtained from the 2019 Indonesian Mortality Table (TMI 2019), which provides age-specific mortality probabilities The mortality data are used to calculate survival and mortality probabilities in the analysis of risk-based gross premiums. The analysis is conducted under the following assumptions: entry age of 25 years, retirement age of 59 years, an interest rate of 5.75% per year, and a sum insured of IDR 1,000,000. The portfolio size ranges from 1 to 10,000,000 insured individuals to examine its effect on the relative risk load. These assumptions provide the basis for the premium calculation and simulation analysis.

The steps taken in this study are:

1. Formulating the initial pension life insurance premium model using equations (1), (12), (14), and (15).
2. Deriving the relative risk load formula using Equations (4), (5), and (8).
3. Deriving the net premium formula using Equation (1).
4. Formulating the risk-based gross premium model using Equation (3).
5. Applying the proposed model using Equations (6), (10), (11), (12), (14), and (15) to evaluate the effect of portfolio size on the relative risk load and gross premium.

3. RESULT & DISCUSSIONS

3.1. Preparing The Initial Equation for Pension Life Insurance Premiums

Like other life insurance products, pension life insurance consists of two main components: premium payments and insurance benefits. In this model, a premium of P is paid by the insured monthly from the entry age (x years) until the retirement age r. The expected present value of the total net premium is calculated using the temporary life annuity defined in Equation (12), assuming monthly premium payments over $r - x$ years:

$$12P\ddot{a}_{x:\overline{r-x}|}^{(12)}$$

The guaranteed benefits consist of two types: pension benefits and pre-pension death benefits. The pension benefit of S is payable monthly starting at the pension age throughout the remaining lifetime of the insured, and according to equation (14), it has a present value expectation as follows:

$$12S_{r-x|}\ddot{a}_x^{(12)}$$

Meanwhile, if the insured dies before pension age, then female beneficiary currently aged y years will receive a benefit of S, paid monthly for as long as they live, commencing at the insured's retirement age. Based on equation (15), the expected present value of this benefit is expressed as follows:

$$12S_{r-x|}\ddot{a}_y^{(12)}$$

Since this benefit is payable only if the insured dies before retirement age, the expected present value must be weighted by the probability of death before age r, namely $_{r-x|}q_x$. Therefore, according to Equation (1),

$$12P\ddot{a}_{x:\overline{r-x}|}^{(12)} = 12S\left[{}_{r-x|}\ddot{a}_x^{(12)} + \left({}_{r-x|}\ddot{a}_y^{(12)}\ {}_{r-x}q_x\right)\right] \quad (16)$$

Assuming a unit benefit ($S = 1$), Equation (16) becomes,

$$12P\ddot{a}_{x:\overline{r-x}|}^{(12)} = 12\left[{}_{r-x|}\ddot{a}_x^{(12)} + \left({}_{r-x|}\ddot{a}_y^{(12)}\ {}_{r-x}q_x\right)\right] \quad (17)$$

Furthermore, Substituting the deferred annuity expressions from Equations (14) and (15) into Equation (17) gives

$$12P\ddot{a}_{x:\overline{r-x}|}^{(12)} = 12\left[v^{r-x}\ {}_{r-x}p_x\ \ddot{a}_r^{(12)} + \left(v^{r-x}\ {}_{r-x}p_y\ \ddot{a}_{y+r-x}^{(12)}\ {}_{r-x}q_x\right)\right] \quad (18)$$

Equation (18) forms the basis for developing the net premium model before incorporating the relative risk load associated with claim uncertainty.

3.2. Derivation of the Relative Risk Load Formula.

The derivation of the relative risk load begins with the formulation of the variance of the insurer's benefit payments. Variance measures the variability of the insurer's future benefit payments around their expected value. In this study, two main scenarios are analyzed:

1. The survival scenario, where the insured survives to pension age and receives lifetime annuity benefits.
2. The death-before-pension scenario, where the insured dies before reaching pension

age, and the annuity benefits are transferred to their heirs.

These two mutually exclusive scenarios exhaust all possible outcomes, so based on equation (8), it can be understood that the total probability satisfies the relationship:

$$ {}_{r-x}p_x + {}_{r-x}q_x = 1 \tag{19}$$

From Equation (18), it follows that:

$$ 12P\ddot{a}^{(12)}_{x:\overline{r-x}|} - 12\left(v^{r-x}\,{}_{r-x}p_x\,\ddot{a}^{(12)}_{r}\right) - 12\left(v^{r-x}\,{}_{r-x}p_y\,\ddot{a}^{(12)}_{y+r-x}\,{}_{r-x}q_x\right) = 0 \tag{20}$$

It follows from Equations (15), (19), and (20) that

$$ 12\left[{}_{r-x}p_x\left(P\ddot{a}^{(12)}_{x:\overline{r-x}|} - v^{r-x}\ddot{a}^{(12)}_{r}\right) + {}_{r-x}q_x\left(P\ddot{a}^{(12)}_{x:\overline{r-x}|} - {}_{r-x|}\ddot{a}^{(12)}_{y}\right)\right] = 0 \tag{21}$$

The first term represents the survival scenario, in which the insured survives until retirement and receives a deferred life annuity: the net premium paid during the working period minus the present value of the lifetime annuity benefit, multiplied by the probability of surviving until pension age. The second term corresponds to the pre-retirement death scenario: the net premium minus the present value of the deferred annuity for heirs, multiplied by the probability of dying during the work period.

The formula for variance is obtained as follows:

$$ \sigma^2 = 144\left[{}_{r-x}p_x\left(P\ddot{a}^{(12)}_{x:\overline{r-x}|} - v^{r-x}\,\ddot{a}^{(12)}_{r}\right)^2 + {}_{r-x}q_x\left(P\ddot{a}^{(12)}_{x:\overline{r-x}|} - {}_{r-x|}\ddot{a}^{(12)}_{y}\right)^2\right] \tag{22}$$

Substituting Equation (14) into Equation (22) gives

$$ \sigma^2 = 144\left[{}_{r-x}p_x\,{}_{r-x}q_x\,(v^{r-x})^2\left(\ddot{a}^{(12)}_{r} - {}_{r-x}p_y\,\ddot{a}^{(12)}_{y+r-x}\right)^2\right] \tag{23}$$

So the standard deviation (σ) can be expressed as:

$$ \sigma = 12\left(v^{r-x}\ddot{a}^{(12)}_{r} - {}_{r-x}p_y\,\ddot{a}^{(12)}_{y+r-x}\sqrt{{}_{r-x}p_x\,{}_{r-x}q_x}\right) \tag{24}$$

Once the standard deviation has been determined, the risk load can be derived. Following Mualem and Zaks (2019), the insurer applies a risk loading equal to three times the standard deviation to provide a safety margin against claim uncertainty. Therefore, based on Equation (4), the risk load is expressed as:

$$ \delta = 12P^*\ddot{a}^{(12)}_{x:\overline{r-x}|} - 12\left[v^{r-x}\,{}_{r-x}p_x\,\ddot{a}^{(12)}_{r} + \left(v^{r-x}\,{}_{r-x}p_y\,\ddot{a}^{(12)}_{y+r-x}\,{}_{r-x}q_x\right)\right] = 3\sigma \tag{25}$$

The relative risk load is defined as the ratio of the risk load to the net premium. Based on equation (5), the relative risk load is formulated as:

$$ \delta^* = \frac{3\left(\left|\ddot{a}^{(12)}_{r} - \frac{{}_{r-x}p_y\,\ddot{a}^{(12)}_{y+r-x}}{v^{r-x}}\right|\sqrt{{}_{r-x}p_x\,{}_{r-x}q_x}\right)}{\left({}_{r-x}p_x\,\ddot{a}^{(12)}_{r} + \left({}_{r-x}p_y\,\ddot{a}^{(12)}_{y+r-x}\,{}_{r-x}q_x\right)\right)} \times 100\% \tag{26}$$

The denominator in equation (26) represents the expected present value (mean) of the insurer's benefit liability. Once the formula for calculating the relative risk load is obtained, the next step is to set up the formula for calculating the net premium. The net premium calculation formula is used as the basis for calculating the gross premium.

3.3. Derivation of The Net Premium Formula

Based on Equations (1) and (16), the net premium is given by

$$ P = \frac{12S\left({}_{r-x|}\ddot{a}^{(12)}_{x} + {}_{r-x|}\ddot{a}^{(12)}_{y} \times {}_{r-x}q_x\right)}{12\ddot{a}^{(12)}_{x:\overline{r-x}|}} \tag{27}$$

After getting the formula for calculating the net premium, the next step is to put together a model for calculating the gross premium.

3.4. Development of The Gross Premium Model

Based on Equations (3), (26), and (27), the calculation model for gross premiums for pension life insurance can be expressed as:

$$ P^* = \frac{12S\left({}_{r-x|}\ddot{a}^{(12)}_{x} + {}_{r-x|}\ddot{a}^{(12)}_{y}\,{}_{r-x}q_x\right)}{12\ddot{a}^{(12)}_{x:\overline{r-x}|}}\left(1 + \frac{3\left(\left|\ddot{a}^{(12)}_{r} - \frac{{}_{r-x}p_y\,\ddot{a}^{(12)}_{y+r-x}}{v^{r-x}}\right|\sqrt{{}_{r-x}p_x\,{}_{r-x}q_x}\right)}{\left({}_{r-x}p_x\,\ddot{a}^{(12)}_{r} + \left({}_{r-x}p_y\,\ddot{a}^{(12)}_{y+r-x}\,{}_{r-x}q_x\right)\right)} \times 100\%\right) \tag{28}$$

3.5. Implementation of Risk-Based Gross Premium Calculation Model in Pension Life Insurance

In this study, the amount of gross pension life insurance premium was calculated based on equation (28). The calculation was based on the 2019 Indonesian Mortality Table (TMI) and an interest rate of 5.75% per year, assuming that both premiums and benefits are payable monthly (m = 12). The monthly pension benefit was set at IDR 1,000,000.

The insured is assumed to start working at age 25 and retire at age 59, giving a working period of 34 years. There are two benefit scenarios analyzed:

1. If the insured survives until retirement age, they will receive a lifelong annuity benefit with monthly payments.
2. If the insured dies before retirement, the lifelong annuity benefit with monthly payments will be given to the beneficiary who is assumed to be 20 years old at the start of the contract.

A simulation was also conducted to analyze the effect of the size of the insured group on the relative risk load and the premium amount. The portfolio size was varied across 1, 10, 100, 1,000, 10,000, 100,000, 1,000,000, and 10,000,000 insured individuals. To evaluate the relative risk load, the actuarial values obtained above are substituted into Equation (26), yielding the following expression:

$$\delta^* = \frac{3\left(\left|\ddot{a}_{59}^{(12)} - \frac{{}_{34}p_{20}\,\ddot{a}_{54}^{(12)}}{v^{34}}\right|\sqrt{{}_{34}p_{25}\;{}_{34}q_{25}}\right)}{\left({}_{34}p_{25}\,\ddot{a}_{59}^{(12)} + \left({}_{34}p_{20}\,\ddot{a}_{54}^{(12)}\;{}_{34}q_{25}\right)\right)} \times 100\%$$

To calculate the value of the relative risk load, we first determine the actuarial values as follows:

1. The probability that the insured (aged 25) survives for 34 years $({}_{34}p_{25})$.
   Based on equations (7) and (8), the value of ${}_{34}p_{25}$ is:
   $${}_{34}p_{25} = \frac{l_{59}}{l_{25}} = \frac{88.457.3}{98.684.2} = 0.896367$$
   Where l_{59} and l_{25} are obtained from the 2019 Indonesian Mortality Table (TMI) for males.
2. The probability that the insured (aged 25) will die within 34 years $({}_{34}q_{25})$.
   Based on equation (8), the value of ${}_{34}q_{25}$ is:
   $${}_{34}q_{25} = 1 - {}_{34}p_{25} = 1 - 0.896367 = 0.103633$$
3. The probability that an heir (aged 20) will survive for 34 years $({}_{34}p_{20})$.
   Based on equations (7) and (8), the value of ${}_{34}p_{20}$ is:
   $${}_{34}p_{20} = \frac{l_{54}}{l_{20}} = \frac{9.496.592}{9.928.714} = 0.956477$$
   with the values of l_{54} and l_{20} obtained from the 2019 Indonesian Mortality Table (TMI) for females.
4. The expected present value of a discrete life annuity for a 59-year-old insured. with payments made at the beginning of each month $\left(\ddot{a}_{59}^{(12)}\right)$. Based on equation (10), $\ddot{a}_{59}^{(12)}$ can be formulated as:
   $$\ddot{a}_{59}^{(12)} = \sum_{t=0}^{\infty}\frac{1}{12}v^{\frac{t}{12}}{}_{\frac{t}{12}}p_{59} = \left(\frac{1}{12}v^{0}\,{}_{0}p_{59}\right) + \left(\frac{1}{12}v^{\frac{1}{12}}{}_{\frac{1}{12}}p_{59}\right) + \cdots = 12.569658$$
5. The expected present value of a discrete whole life annuity for a 54-year-old beneficiary with payments made at the beginning of each month $\left(\ddot{a}_{54}^{(12)}\right)$. Based on equation (11), $\ddot{a}_{54}^{(12)}$ can be formulated as:
   $$\ddot{a}_{54}^{(12)} = \sum_{t=0}^{\infty}\frac{1}{12}v^{\frac{t}{12}}{}_{\frac{t}{12}}p_{20+59-25} = \sum_{t=0}^{\infty}\frac{1}{12}v^{\frac{t}{12}}{}_{\frac{t}{12}}p_{54} = 14.062180$$

Based on the actuarial values obtained, the relative risk load is:

$$\delta^* = \frac{3\left(\left|12.569658 - \frac{0.956477 \times 14.062180}{0.149440}\right|\sqrt{0.896367 \times 0.103633}\right)}{\left(0.896367 \times 12.569658 + (0.956477 \times 14.062180 \times 0.103633)\right)} = \frac{3 \times 23.600643}{12.660906} = 5.592169 = 559.216\%$$

Equation (6) is used to evaluate the effect of portfolio size on the relative risk load. The calculation results for various group sizes are presented in Table 1.

Table 1. Relative Risk Load for Different Portfolio Sizes.

Size of the Insured Group (N) (People)	Relative Risk Load (%)
1	559.216
10	176.839
100	55.921
1,000	17.683
10,000	5.592
100,000	1.768
1,000,000	0.559
10,000,000	0.176

After determining the relative risk load for various insured group sizes, the next step is to calculate the net premium as the basis for determining the gross premium. To calculate the net premium value, the actuarial values are first determined as follows:

1. The present value expectation of a discrete-term life annuity starting at age 25 with payments made at the beginning of each month, lasting for 59-25 years $\left(\ddot{a}^{(12)}_{25:\overline{59-25}|}\right)$ Using equation (12), then $\ddot{a}^{(12)}_{25:\overline{59-25}|}$ is formulated as:

$$\ddot{a}^{(12)}_{25:\overline{59-25}|} = \ddot{a}^{(12)}_{25:\overline{34}|} = \sum_{t=0}^{12(34)-1} \frac{1}{12} v^{\frac{t}{12}} {}_{\frac{t}{12}}p_{25} = \sum_{t=0}^{407} \frac{1}{12} v^{\frac{t}{12}} {}_{\frac{t}{12}}p_{25} = 15.007265$$

2. The expected present value of a discrete life annuity with payments made at the beginning of each month, starting after a 34-year deferment, for a person who is currently 25 years old $\left({}_{34|}\ddot{a}^{(12)}_{25}\right)$.

   Using equation (14), the value of ${}_{34|}\ddot{a}^{(12)}_{25}$ is:

$${}_{34|}\ddot{a}^{(m)}_{25} = v^{34}\,{}_{34}p_{25}\ddot{a}^{(m)}_{59} = \left(\frac{1}{1+0.0575}\right)^{34} \times 0.896367 \times 12.569658 = 1.683744$$

3. The expected present value of a discrete life annuity with payments made at the beginning of each month, starting after a delay of 34 years, for a beneficiary who is currently 20 years old $\left({}_{34|}\ddot{a}^{(12)}_{20}\right)$.

   Based on equation (15), the value of ${}_{34|}\ddot{a}^{(12)}_{20}$ is:

$${}_{34|}\ddot{a}^{(m)}_{20} = v^{34}\,{}_{34}p_{20}\ddot{a}^{(m)}_{54} = \left(\frac{1}{1+0.0575}\right)^{34} \times 0.956477 \times 14.062180 = 2.009990$$

Substituting the actuarial values $\ddot{a}^{(12)}_{25:\overline{34}|}$, ${}_{34|}\ddot{a}^{(12)}_{25}$, and ${}_{34|}\ddot{a}^{(m)}_{20}$ into Equation (27) yields:

$$P = \frac{12\times1{,}000{,}000\left(1.683744+(2.009990\times0.103633)\right)}{12\times15.007265} = 126{,}070.25$$

After calculating the net premium, the next step is to calculate the gross premium for different sizes of insured groups. Based on equation (28), the results of the gross premium calculations for various group sizes are presented in Table 2.

Table 2. Gross Premium by Portfolio Size

Size of the Insured Group (N) (People)	Gross Premium (IDR)
1	831,075.25
10	349,011.61
100	196,596.99
1,000	148,363.25
10,000	128,599.21
100,000	128,299.17
1,000,000	126,774.98
10.000,000	126,292.13

Based on the results presented in Tables 1 and 2, the relative risk load decreases as the portfolio size increases. This occurs because the insurance risk is diversified across a larger number of insured individuals, thereby reducing the variability of aggregate claims. Consequently, the gross premium also decreases and approaches the net premium. For small portfolios, the difference between the gross and net premiums is substantial due to the relatively high-risk load. In contrast, for large portfolios, this difference becomes negligible. These findings are consistent with the law of large numbers, which states that the variability of the average claim decreases as the portfolio size increases. In contrast, the net premium remains unchanged across all portfolio sizes because it depends solely on the expected present value of future benefits and is therefore independent of the number of insured individuals.

4. CONCLUSION AND RECOMMENDATIONS

This study developed a risk-based gross premium model for pension life insurance using a variance- and portfolio-based approach. The proposed model, as presented in Equation (28), determines the gross premium by adding a relative risk load to the net premium. The relative risk load is derived from the standard deviation and the expected present value of future benefits, enabling the premium to reflect the uncertainty associated with future claim payments. The numerical results demonstrate that the relative risk load decreases as the portfolio size increases because claim variability is reduced through risk diversification. Consequently, the gross premium approaches the net premium for larger portfolios. In contrast, the net premium remains unchanged

regardless of portfolio size, since it depends solely on the expected present value of future benefits. These findings are consistent with the law of large numbers, which states that the variability of the average claim decreases as the number of insured individuals increases. Future research may extend the proposed model by considering pension life insurance products that provide annuity benefits immediately to the beneficiary if the insured dies before reaching retirement age. In addition, the model may be expanded to incorporate disability retirement and early retirement benefits, thereby accommodating a wider range of benefit contingencies and improving its applicability in pension life insurance practice.

DAFTAR PUSTAKA

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